# חATIBIA UחIVERSITY <br> OF SCIEПCE AПD TECHחOLOGY <br> FACULTY OF HEALTH AND APPLIED SCIENCES 

DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BSAM | LEVEL: 7 |
| COURSE CODE: MMO702S | COURSE NAME: MATHEMATICAL MODELLING 2 |
| SESSION: NOVEMBER 2022 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 298 (To be Converted to 100\%) |


| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER | PROF. S. A. REJU |
| MODERATOR: | PROF. O. D. MAKINDE |

## INSTRUCTIONS

1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencil.
3. Use of COMMA is not allowed as a DECIMAL POINT.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

## QUESTION 1 [125 MARKS]

(a) Define the Linear Congruential Generator (LCG), and using a seed 2, multiplier 15, increment 3 and modulus 85 , obtain the sequence of pseudo-random numbers using the LCG.
(33 Marks)

Is there cycling? (YES/NO). If so, when does it occur?
(3 Marks)
(b) A data communication line delivers a block of information every 10 microseconds ( $\mu \mathrm{s}$ ). A decoder checks each block for errors and corrects the errors if necessary. It takes $1 \mu \mathrm{~s}$ to determine whether the block has any errors. If the block has one error it takes $5 \mu \mathrm{~s}$ to correct it and if it has more than 1 error it takes $20 \mu$ s to correct the error. Blocks wait in the queue when the decoder falls behind. Suppose that the decoder is initially empty and that the number of errors in the first 16 blocks are: $0,1,4,1,0,4,0,1,0,3,1,2,1,2,1$, 4,

Construct a simulation table for the queuing model, showing arrival times, number of errors, waiting, service and departure times.
(80 Marks)
(c) From your simulation table in (b), determine the following performance measures (correct to 2 decimal places for non-integer numbers):
(12 Marks, 2 Marks each)
(i) Average number of blocks in the system
(ii) Average block waiting time
(iii) Maximum simulation time
(iv) Decoder busy duration
(v) Decoder utilization time
(vi) Decoder idle time

## QUESTION 2 [40 MARKS]

(a) A small-scale vocational business firm produces two farming implements: hoes and shovels and realises a net unit profit of $N \$ 125$ per hoe and $N \$ 140$ per shovel. Assume that the firm has up to 250 square metres of iron sheet and 200 metres of wood length to devote to a small farming project plus a signed contract of supplying 10 hoes and 15 shovels to a family farm during the period of the project. Moreover, assume that it requires 2 square metres of iron and 0.65 metre of wood to fabricate a hoe and 3 square metres of iron and 0.85 metre of wood to produce a shovel. Formulate and solve the model for maximising the firm's profits for hoes and shovels.
(20 Marks)
(b) (i) Define post-optimality analysis for linear optimisation problems
(5 Marks)
(ii) Discuss the analysis for change in the firm's profits on hoes, showing all expressions to support your conclusion.
(15 Marks)

## QUESTION 3 [40 MARKS]

(a) A spring with a mass of 2 kg has natural length 0.5 m and a force of 25.6 N is required to maintain it stretched to a length of 0.7 m and then released with initial velocity 0 . Formulate an appropriate model equation and solve to obtain the expression for the position of the mass at any time $t$, stating all physical laws to support the fundamental equations and associated concepts of your model and its solution before using the given data.
(25 Marks)
(b) Then suppose that the mass-spring system in (a) is immersed in a fluid with damping constant $c=40$. Stating the general model differential equations for the damped system, find the position of the mass at any time $t$ if it starts from the equilibrium position and is given a push to start it with an initial velocity of $0.6 \mathrm{~m} / \mathrm{s}$. (15 Marks)

## QUESTION 4 [40 MARKS]

(a) Consider a pottery company that produces bowls and mugs and assume that that per unit profit contribution for bowls is given by ( $\$ 4-0.1 x_{1}$ ) and that per unit profit contribution for mugs is given by ( $\$ 5-0.2 x_{2}$ ).
(i) Formulate a nonlinear profit maximization problem subject to just a labour constraint given by $x_{1}+2 x_{2}=40$ hours
(19 Marks)
(ii) Solve the nonlinear optimisation problem in (i) using the Substitution Method (19 Marks)
(b) Consider a general $2^{\text {nd }}$ degree polynomial

$$
f(x)=a_{3} x^{2}+a_{2} x+a_{1}
$$

(i) State the normal equations for determining the regression coefficients $a_{1}, a_{2}$ and $a_{3}$ of the polynomial $f(x)$ for fitting a set of data.
(6 Marks)
(ii) Consider the following data

| x | 1.2 | 1.5 | 2.0 | 2.6 | 3.2 | 4.5 | 5.2 | 5.7 | 6.0 | 6.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 3.4 | 4.1 | 3.2 | 4.5 | 2.5 | 5.2 |

- Obtain the normal equations for $f(x)$ defined by (a) above using the above data.
- State the 3 -line MATLAB commands for solving the system of three equations (without determining the values of the regression coefficients).

